CHAPTER 4 PROBLEMS AND EXERCISES SOLUTIONS

Table 1 Generic aluminum alloy (E = 70 GPa, ν = 0.33) stiffness matrix and density

$$\mathbf{C}' = \begin{bmatrix} 103.7 & 51.1 & 51.1 & 0 & 0 & 0 \\ 51.1 & 103.7 & 51.1 & 0 & 0 & 0 \\ 51.1 & 51.1 & 103.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 26.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 26.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 26.3 \end{bmatrix} \text{GPa}, \qquad \rho = 2700 \text{ kg/m}^3$$

Table 2 7075-T6 aluminum alloy (E = 71.7 GPa, ν = 0.33) stiffness matrix and density

$$\mathbf{C}' = \begin{bmatrix} 106.2 & 52.3 & 52.3 & 0 & 0 & 0 \\ 52.3 & 106.2 & 52.3 & 0 & 0 & 0 \\ 52.3 & 52.3 & 106.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 26.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 26.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 26.7 \end{bmatrix} \text{GPa}, \qquad \rho = 2810 \text{ kg/m}^3$$

Table 3 T300-914 CFRP unidirectional composite stiffness matrix and density

$$\mathbf{C}' = \begin{bmatrix} 143.8 & 6.2 & 6.2 & 0 & 0 & 0 \\ 6.2 & 13.3 & 6.5 & 0 & 0 & 0 \\ 6.2 & 6.5 & 13.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.7 \end{bmatrix} \text{GPa}, \qquad \rho = 1560 \text{ kg/m}^3$$

Table 4 Fully orthotropic CFRP composite stiffness matrix and density

$$\mathbf{C}' = \begin{bmatrix} 70 & 23.9 & 6.2 & 0 & 0 & 0 \\ 23.9 & 33 & 6.8 & 0 & 0 & 0 \\ 6.2 & 6.8 & 14.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 21.9 \end{bmatrix} \text{GPa}, \qquad \rho = 1500 \text{ kg/m}^3$$

PROBLEM 1: CHRISTOFFEL EQUATION FOR BULK WAVES

Consider a plane wave propagating through an isotropic material of stiffness matrix $\mathbf{C}$ in a direction contained in the vertical plane $x_1 O x_3$ and rotated by an angle θ about the x_2 axis. Do the following:

(a) Calculate the stiffness tensor $\mathbf{c}$
(b) Calculate the acoustic tensor $\mathbf{\Gamma}$
(c) State the Christoffel equation and find its eigenvalues
(d) Find the wavespeeds corresponding to these eigenvalues
(e) Find the polarization direction for each wavespeed
(f) Identify any pure waves that might show up during the analysis

PROBLEM 2: CHRISTOFFEL EQUATION IN BULK ISOTROPIC MATERIAL

The Christoffel equation was developed for anisotropic materials. However, in this problem, we will apply the Christoffel equation approach to an isotropic material in order to establish that the formalism developed for wave propagation in anisotropic materials can be also used for wave propagation in isotropic materials and gives the expected results.

Consider a plane wave propagating through an isotropic material of elastic constants E and ν. The wave propagates in a direction contained in the vertical plane $x_1 O x_3$ and rotated by an angle θ about the x_2 axis. Do the following:

(a) Calculate the stiffness matrix $\mathbf{C}$ and the stiffness tensor $\mathbf{c}$
(b) Calculate the acoustic tensor $\mathbf{\Gamma}$
(c) Find the eigenvalues of the Christoffel equation
(d) Find the wavespeeds corresponding to these eigenvalues
(e) Find the polarization direction for each wavespeed
(f) Identify any pure waves that might show up during the analysis
(g) Discuss your results

<u>Numerical values</u>:
Wavefront direction angle $\theta = 0°, 10°, 15°, 30°, 45°, 60°, 75°, 90°$.
Material: 7075-T6 aluminum, Table 2

PROBLEM 3: CHRISTOFFEL EQUATION IN BULK UNIDIRECTIONAL COMPOSITE

Consider a plane wave propagating through an unidirectional composite material of stiffness matrix $\mathbf{C}$. The wave propagates in the vertical plane $x_1 O x_3$ and rotated by an angle θ about the x_2 axis. Do the following:

(a) Calculate the stiffness tensor $\mathbf{c}$
(b) Calculate the acoustic tensor $\mathbf{\Gamma}$
(c) Find the eigenvalues of the Christoffel equation
(d) Find the wavespeeds corresponding to these eigenvalues
(e) Find the polarization direction for each wavespeed
(f) Identify any pure waves that might show up during the analysis
(g) Discuss your results

Numerical values:
Wavefront direction angle $\theta = 0°, 10°, 15°, 30°, 45°, 60°, 75°, 90°$.
T300/914 CFRP, Table 3

PROBLEM 4: CHRISTOFFEL EQUATION IN BULK ORTHOTROPIC COMPOSITE

Consider a plane wave propagating through an unidirectional composite material of stiffness matrix $\mathbf{C}$. The wave propagates in the vertical plane $x_1 O x_3$ and rotated by an angle θ about the x_2 axis. Do the following:

(a) Calculate the stiffness tensor $\mathbf{c}$
(b) Calculate the acoustic tensor $\mathbf{\Gamma}$
(c) Find the eigenvalues of the Christoffel equation
(d) Find the wavespeeds corresponding to these eigenvalues
(e) Find the polarization direction for each wavespeed
(f) Identify any pure waves that might show up during the analysis
(g) Discuss your results

Numerical values:
Wavefront direction angle $\theta = 0°, 10°, 15°, 30°, 45°, 60°, 75°, 90°$.
Fully orthotropic CFRP, Table 4

PROBLEM 5: LAMINA CHRISTOFFEL EQUATION FOR ISOTROPIC LAMINA

Given an isotropic lamina of thickness *h*, solve the lamina Christoffel equation for guided wave propagation along the fiber direction x_1. Do the following:

(a) For a given wavespeed v, set up the cubic equation α^2, where α^2 are the partial-wave slowness coefficients

(b) Solve the cubic equation, find the three α^2 roots $\alpha_I^2, \alpha_{II}^2, \alpha_{III}^2$, and the corresponding six eigenvalues $\alpha^{(1)} = +\alpha_I$, $\alpha^{(2)} = -\alpha_I$, $\alpha^{(3)} = +\alpha_{II}$, $\alpha^{(4)} = -\alpha_{II}$, $\alpha^{(5)} = +\alpha_{III}$, $\alpha^{(6)} = -\alpha_{III}$

(c) Substitute each eigenvalue $\alpha^{(i)}, i = 1,...,6$ into the Christoffel matrix and find the corresponding eigenvectors $\mathbf{U}^{(i)}$, $i = 1,...,6$

(d) Repeat for other values of v

(e) Discuss your findings

Numerical example: 7075-T6 aluminum alloy, Table 2

PROBLEM 6: LAMINA CHRISTOFFEL EQUATION FOR UNIDIRECTIONAL COMPOSITE LAMINA

Given a unidirectional composite lamina of thickness *h*, solve the lamina Christoffel equation for guided wave propagation along the fiber direction x_1. Do the following:

(a) For a given wavespeed v, set up the cubic equation α^2, where α^2 are the partial-wave slowness coefficients

(b) Solve the cubic equation, find the three α^2 roots $\alpha_I^2, \alpha_{II}^2, \alpha_{III}^2$, and the corresponding six eigenvalues $\alpha^{(1)} = +\alpha_I$, $\alpha^{(2)} = -\alpha_I$, $\alpha^{(3)} = +\alpha_{II}$, $\alpha^{(4)} = -\alpha_{II}$, $\alpha^{(5)} = +\alpha_{III}$, $\alpha^{(6)} = -\alpha_{III}$

(c) Substitute each eigenvalue $\alpha^{(i)}, i = 1,...,6$ into the Christoffel matrix and find the corresponding eigenvectors $\mathbf{U}^{(i)}$, $i = 1,...,6$

(d) Repeat for other values of v

(e) Discuss your findings

Numerical example: T300/914 CFRP, Table 3

PROBLEM 7: LAMINA CHRISTOFFEL EQUATION FOR OFF-AXIS COMPOSITE LAMINA

Given a unidirectional composite lamina of thickness *h*, solve the lamina Christoffel equation for guided wave propagation along an off-axis direction. Do the following:

(a) For a given wavespeed v, set up the cubic equation α^2, where α^2 are the partial-wave slowness coefficients
(b) Solve the cubic equation, find the three α^2 roots $\alpha_I^2, \alpha_{II}^2, \alpha_{III}^2$, and the corresponding six eigenvalues $\alpha^{(1)} = +\alpha_I$, $\alpha^{(2)} = -\alpha_I$, $\alpha^{(3)} = +\alpha_{II}$, $\alpha^{(4)} = -\alpha_{II}$, $\alpha^{(5)} = +\alpha_{III}$, $\alpha^{(6)} = -\alpha_{III}$
(c) Substitute each eigenvalue $\alpha^{(i)}, i = 1,...,6$ into the Christoffel matrix and find the corresponding eigenvectors $\mathbf{U}^{(i)}$, $i = 1,...,6$
(d) Repeat for other values of v
(e) Repeat (a) through (e) for other values of θ
(f) Discuss your findings

Numerical example: T300/914 CFRP, Table 3;
Off-axis direction angle $\theta = 0°, 5°, ..., 85°, 90°$, i.e., in $5°$ increments.

PROBLEM 8: LAMINA CHRISTOFFEL EQUATION IN ORTHOTROPIC COMPOSITE LAMINA

Given a fully orthotropic composite lamina of given thickness *h*, solve the lamina Christoffel equation for guided wave propagation along the fiber direction. Do the following:

(a) For a given wavespeed v, set up the cubic equation in partial wave slowness coefficients α^2
(b) Solve the cubic equation, find the three α^2 roots $\alpha_I^2, \alpha_{II}^2, \alpha_{III}^2$, and the corresponding six eigenvalues $\alpha^{(1)} = +\alpha_I$, $\alpha^{(2)} = -\alpha_I$, $\alpha^{(3)} = +\alpha_{II}$, $\alpha^{(4)} = -\alpha_{II}$, $\alpha^{(5)} = +\alpha_{III}$, $\alpha^{(6)} = -\alpha_{III}$
(c) Substitute each eigenvalue $\alpha^{(i)}, i = 1,...,6$ into the Christoffel matrix and find the corresponding eigenvectors $\mathbf{U}^{(i)}$, $i = 1,...,6$
(d) Repeat for other values of v
(e) Discuss your findings

Numerical example: Fully orthotropic CFRP composite, Table 4